Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA14/01**

Mathematics

International Advanced Level
Pure Mathematics P4

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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1. The curve C has equation

$$xy^2 = x^2y + 6 \qquad x \neq 0 \quad y \neq 0$$

Find an equation for the tangent to C at the point $P(2, 3)$, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(6)

★ Implicit differentiation:

When differentiating y, multiply by $\frac{dy}{dx}$

$$xy^2 = x^2y + 6$$

We also have multiplications ∴ Product Rule

$$\frac{d}{dx}(uv) = uv' + u'v$$

$$y^2 + 2yx\frac{dy}{dx} = 2xy + x^2\frac{dy}{dx} \quad \text{M1 dM1 A1}$$

$$y^2 - 2xy = x^2\frac{dy}{dx} - 2yx\frac{dy}{dx} \quad \text{collect } \frac{dy}{dx} \text{ on one side}$$

$$y^2 - 2xy = \frac{dy}{dx}(x^2 - 2xy)$$

$$\frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy}$$

Substitute $P(2,3)$ to get $\frac{dy}{dx}$ at P:

$$\frac{dy}{dx} = \frac{3^2 - 2(2)(3)}{2^2 - 2(2)(3)}$$

$$= \frac{9-12}{4-12} = \frac{-3}{-8}$$

$$= \frac{3}{8} \quad \text{M1}$$

Use $y - y_1 = m(x - x_1)$ to get the equation of the tangent:

$$y - 3 = \frac{3}{8}(x-2) \quad \text{dM1}$$

$$8y - 24 = 3x - 6$$

$$0 = 3x - 8y + 18 \quad \text{A1}$$

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Question 1 continued

Q1

(Total 6 marks)

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2. (a) Find, in ascending powers of x, the first three non-zero terms of the binomial series expansion of

$$\sqrt[3]{1+4x^3} \qquad |x| < \frac{1}{\sqrt[3]{4}}$$

giving each coefficient as a simplified fraction.

(4)

(b) Use the expansion from part (a) with $x = \frac{1}{3}$ to find a rational approximation to $\sqrt[3]{31}$

(3)

(a) Get the formula for the Binomial Expansion from the formula booklet:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots, \text{ Range of validity } |x|<1$$

Substitute into the Formula:

$$(1+4x^3)^{\frac{1}{3}} = 1 + \frac{1}{3}(4x^3) + \frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(4x^3)^2 + \dots \quad \text{M1}$$

$$= 1 + \frac{4}{3}x^3 + \frac{-\frac{2}{9}}{2}(16x^6) + \dots \quad \text{M1}$$

$$= 1 + \frac{4}{3}x^3 - \frac{16}{9}x^6 + \dots \quad \text{A1A1}$$

(b) $1 + 4(\frac{1}{3})^3 = 1 + 4 \cdot \frac{1}{27} = \frac{31}{27}$ M1

We are not asked to find $\sqrt[3]{\frac{31}{27}}$, but $\sqrt[3]{31}$, hence we need to get rid of $\sqrt[3]{\frac{1}{27}} = \frac{1}{3}$ by multiplying our expansion by 3:

$$3 \times \sqrt[3]{\frac{31}{27}} = 3\left(1 + \frac{4}{3}\left(\frac{1}{3}\right)^3 - \frac{16}{9}\left(\frac{1}{3}\right)^6\right) \approx \sqrt[3]{31} \quad \text{dM1}$$

$$\therefore \sqrt[3]{31} \approx \frac{6869}{2187} \quad \text{A1}$$

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Question 2 continued

Q2

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3. The curve C has parametric equations

$$x = 3 + 2\sin t \qquad y = \frac{6}{7 + \cos 2t} \qquad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$$

(a) Show that C has Cartesian equation

$$y = \frac{12}{(7 - x)(1 + x)} \qquad p \leqslant x \leqslant q$$

where p and q are constants to be found.

(6)

(b) Hence, find a Cartesian equation for C in the form

$$y = \frac{a}{x + b} + \frac{c}{x + d} \qquad p \leqslant x \leqslant q$$

where a, b, c and d are constants.

(3)

(a) Rearrange $x=$ to make $\sin t$ the subject

$$x = 3 + 2\sin t$$

$$\frac{x-3}{2} = \sin t \quad \text{M1}$$

Change $y = \ldots$ to get a $\sin t$ term:

$$y = \frac{6}{7 + \cos 2t}$$

$$= \frac{6}{7 + (1 - 2\sin^2 t)} \quad \text{M1}$$

use double angle formula: $\cos 2A = 1 - 2\sin^2 A$

$$= \frac{6}{8 - 2\sin^2 t}$$

Substitute in $\sin t = \frac{x-3}{2}$:

$$y = \frac{6}{8 - 2\left(\frac{x-3}{2}\right)^2} \quad \text{A1}$$

$$= \frac{6}{8 - 2\cdot\frac{(x-3)^2}{4}} = \frac{24}{32 - 2(x-3)^2}$$

$$= \frac{12}{16 - (x-3)^2} = \frac{12}{(4 - x + 3)(4 + x - 3)}$$

$$= \frac{12}{(7-x)(x+1)} \quad \text{M1A1}$$

To get the domain use the x parameter:

$x_{min} = 3 + 2\sin(-\frac{\pi}{2}) = 1 = p$

$x_{max} = 3 + 2\sin(\frac{\pi}{2}) = 5 = q$ $\quad \therefore$ Domain: $1 \leqslant x \leqslant 5$ B1

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Question 3 continued

(b) We are asked to do Partial Fractions

$$\frac{12}{(7-x)(x+1)} \equiv \frac{A}{(7-x)} + \frac{B}{(x+1)}$$

Manipulate this to get common denominator

$$\frac{12}{(7-x)(x+1)} \equiv \frac{A(x+1)+B(7-x)}{(7-x)(x+1)}$$

Equate the numerators and substitute in values in order to get A, B

$12 = A(x+1)+B(7-x)$

choose values to substitute in so that brackets become 0, to make your life easier!

$x=-1 \Rightarrow 12 = A(\cancel{-1+1}^{0}) + B(7--1)$

$12 = 8B \Rightarrow B = \frac{3}{2}$

$x=7 \Rightarrow 12 = A(7+1) + B(\cancel{7-7}^{0})$

$12 = 8A \Rightarrow A = \frac{3}{2}$ M1

$y = \frac{3}{2(x+1)} + \frac{3}{2(7-x)}$ is not the right form: $\frac{3}{2(x+1)} + \frac{3}{-1 \cdot 2(x-7)} = \frac{3}{2(x+1)} - \frac{3}{2(x-7)}$

the instruction wants this x to be positive.

$\therefore y = \frac{3}{2(x+1)} - \frac{3}{2(x-7)}$ A1 A1

Question 3 continued

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Question 3 continued

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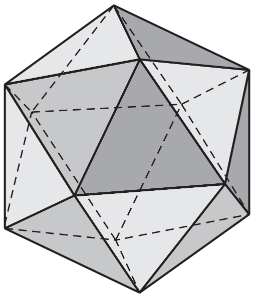

Figure 1

A regular icosahedron of side length x cm, shown in Figure 1, is expanding uniformly.

The icosahedron consists of 20 congruent equilateral triangular faces of side length x cm.

(a) Show that the surface area, A cm^2, of the icosahedron is given by

$$A = 5\sqrt{3}x^2$$

(2)

Given that the volume, V cm^3, of the icosahedron is given by

$$V = \frac{5}{12}\left(3+\sqrt{5}\right)x^3$$

(b) show that $\dfrac{dV}{dA} = \dfrac{\left(3+\sqrt{5}\right)x}{8\sqrt{3}}$

(3)

The surface area of the icosahedron is increasing at a constant rate of 0.025 cm^2 s^{-1}

(c) Find the rate of change of the volume of the icosahedron when $x = 2$, giving your answer to 2 significant figures.

(3)

(a) The area of one face can be found using formula:

$\text{area} = \frac{1}{2}ab\sin C$

In this case $a=b=x$ as the triangles are equilateral, also $C=60°$.

Hence: $\text{area} = \frac{1}{2}\cdot x\cdot x\cdot\sin 60°$

$= \frac{\sqrt{3}}{4}x^2$. M1

The whole thing has 20 sides, ∴

$A = 20\cdot\frac{\sqrt{3}}{4}x^2$

$= 5\sqrt{3}x^2$ hence shown

A1

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Question 4 continued

(b) We can find $\frac{dV}{dx}$ from the formula for volume given:

$$V = \frac{5}{12}(3+\sqrt{5})x^3$$

$$\frac{dV}{dx} = \frac{15}{12}(3+\sqrt{5})x^2 \quad \text{M1}$$

We can also get $\frac{dA}{dx}$:

$$A = 5\sqrt{3}x^2$$

$$\frac{dA}{dx} = 10\sqrt{3}x$$

To get $\frac{dV}{dA}$:

$$\frac{dV}{dA} = \frac{dV}{dx} \cdot \frac{dx}{dA} \quad \text{M1}$$

$$= \frac{15}{12}(3+\sqrt{5})x^2 \cdot \frac{1}{10\sqrt{3}x}$$

$$= \frac{\cancel{15}(3+\sqrt{5})x^{\cancel{2}\,1}}{\cancel{120}\,8\sqrt{3}\cancel{x}}$$

$$\frac{dV}{dA} = \frac{(3+\sqrt{5})x}{8\sqrt{3}}$$ hence shown

A1

(c) We are asked to find $\frac{dV}{dt}$.

We are given that $\frac{dA}{dt} = 0.025$ B1

$$\frac{dV}{dt} = \frac{dA}{dt} \cdot \frac{dV}{dA}$$ → found in (b)

$$= 0.025 \cdot \frac{(3+\sqrt{5})x}{8\sqrt{3}} \quad \text{M1}$$

At $x = 2$:

$$\frac{dV}{dt} = 0.025 \cdot \frac{(3+\sqrt{5})(2)}{8\sqrt{3}}$$ plug this into your calculator

$$= 0.019 \text{ cm}^3\text{s}^{-1} \quad \text{A1}$$

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Question 4 continued

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Question 4 continued

Q4

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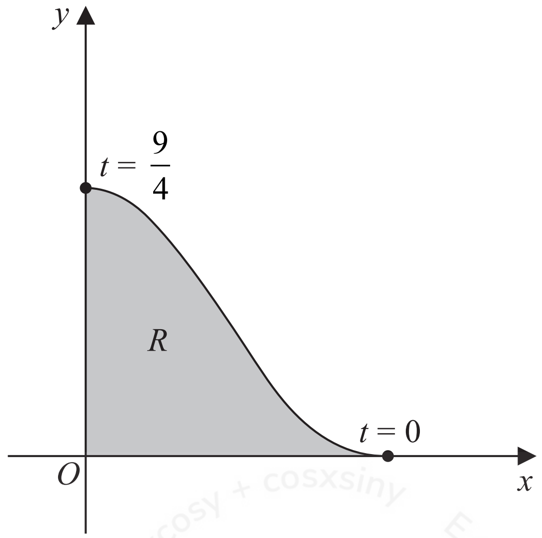

Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = \sqrt{9 - 4t} \qquad y = \frac{t^3}{\sqrt{9 + 4t}} \qquad 0 \leqslant t \leqslant \frac{9}{4}$$

The curve touches the x-axis when $t = 0$ and meets the y-axis when $t = \frac{9}{4}$

The region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the y-axis.

(a) Show that the area of R is given by

$$K\int_0^{\frac{9}{4}} \frac{t^3}{\sqrt{81 - 16t^2}}\,dt$$

where K is a constant to be found.

(4)

(b) Using the substitution $u = 81 - 16t^2$, or otherwise, find the exact area of R.

(*Solutions relying on calculator technology are not acceptable.*)

(6)

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Question 5 continued

(a) ★ Parametric Integration

area $= \int_a^b y \frac{dx}{dt} \, dt$

$$\text{area} = \int_{\frac{9}{4}}^{0} \left(\frac{t^3}{\sqrt{9+4t}} \right) \cdot \frac{d}{dt} \left((9-4t)^{\frac{1}{2}} \right) dt$$

$$= \int_{\frac{9}{4}}^{0} \left(\frac{t^3}{\sqrt{9+4t}} \right) \cdot \left(\frac{1}{2}(9-4t)^{-\frac{1}{2}} \cdot -4 \right) dt$$

★chain rule: multiply by the derivative of the bracket!

$$= \int_{\frac{9}{4}}^{0} \left(\frac{t^3}{\sqrt{9+4t}} \right) \cdot \left(-\frac{2}{\sqrt{9-4t}} \right) dt \quad \text{M1M1}$$

$$= \int_{\frac{9}{4}}^{0} \frac{-2t^3}{\sqrt{(9+4t)(9-4t)}} \, dt \quad \text{ddM1}$$

$$= -2 \int_{\frac{9}{4}}^{0} \frac{t^3}{\sqrt{81-16t^2}} \, dt$$

$$= 2 \int_{0}^{\frac{9}{4}} \frac{t^3}{\sqrt{81-16t^2}} \, dt \quad \text{A1}$$

We can flip the limits by multiplying the integral by -1.

hence shown

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Question 5 continued

(b) To use the given substitution $u = 81 - 16t^2$, we need to find new limits and $dt = \ldots$

New Limits:

t	u
$\frac{9}{4}$	0
0	81

To get dt, differentiate $u = 81 - 16t^2$:

$$\frac{du}{dt} = -32t$$

$$dt = \frac{du}{-32t} \quad \text{B1}$$

Get $t = \ldots$:

$$u = 81 - 16t^2$$

$$\sqrt{\frac{u - 81}{-16}} = t$$

$$t = \sqrt{\frac{81-u}{16}}$$

Substitute into the Integration:

$$\int_{81}^{0} \frac{\left(\sqrt{\frac{81-u}{16}}\right)^3}{\sqrt{u}} \cdot \frac{1}{-32\left(\sqrt{\frac{81-u}{16}}\right)} \, du \quad \text{M1}$$

$$= \int_{81}^{0} \frac{\left(\frac{1}{4}\right)^3 \cdot \left(\sqrt{81-u}\right)^{\cancel{3}\,2}}{\sqrt{u}} \cdot \frac{1}{-32 \cdot \frac{1}{4} \cdot \cancel{\sqrt{81-u}}} \, du$$

$$= \int_{81}^{0} \frac{\frac{1}{8} \cdot (81-u)}{\sqrt{u}} \cdot \frac{1}{-8} \, du$$

$$= \int_{81}^{0} \frac{81-u}{-64\sqrt{u}} \, du$$

$$= -\frac{1}{64} \int_{81}^{0} \frac{81}{u^{\frac{1}{2}}} - \frac{u}{u^{\frac{1}{2}}} \, du$$

$$= -\frac{1}{64} \int_{81}^{0} 81u^{-\frac{1}{2}} - u^{\frac{1}{2}} \, du \quad \text{A1}$$

$$= -\frac{1}{64} \left[\frac{81u^{\frac{1}{2}}}{\frac{1}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right]_{81}^{0} \quad \text{M1}$$

$$= -\frac{1}{64} \left[162u^{\frac{1}{2}} - \frac{2}{3}u^{\frac{3}{2}} \right]_{81}^{0}$$

$$= -\frac{1}{64} \left(\overbrace{\cancel{\left(162(0)^{\frac{1}{2}} - \frac{2}{3}(0)^{\frac{3}{2}}\right)}}^{0} - \left(162(81)^{\frac{1}{2}} - \frac{2}{3}(81)^{\frac{3}{2}}\right) \right) \quad \text{ddM1}$$

$$= -\frac{1}{64}\left(-162 \cdot 9 + \frac{2}{3} \cdot 729\right)$$

$$= -\frac{1}{64}(486 - 1458)$$

$$= \frac{243}{64} \quad \text{A1}$$

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Question 5 continued

Q5

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6. Three **consecutive** terms in a sequence of real numbers are given by

$$k, \quad 1 + 2k \quad \text{and} \quad 3 + 3k$$

where k is a constant.

Use proof by contradiction to show that this sequence is not a geometric sequence.

(5)

Make an Assumption:

"Assume that the sequence is geometric"

We want our assumption to be the opposite of what we are trying to prove.

B1

In the Proof, we are trying to contradict the assumption

In a geometric sequence, quotients of subsequent terms are constant (r).

$$\therefore \; r = \frac{1+2k}{k} = \frac{3+3k}{1+2k}$$ M1

Solve for k:

$$(1+2k)^2 = k(3+3k)$$

$$1 + 4k + 4k^2 = 3k + 3k^2$$

$$k^2 + k + 1 = 0$$ A1

Use the Discriminant:

disc = $b^2 - 4ac$

$$1^2 - 4(1)(1)$$

$$= 1 - 4$$

$$= -3 < 0$$ dM1

Since the discriminant is less than 0, there are no real solutions for k.

This means the sequence can't be geometric which is a contradiction to our assumption.

Hence proven by contradiction that the sequence is not geometric. A1

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Question 6 continued

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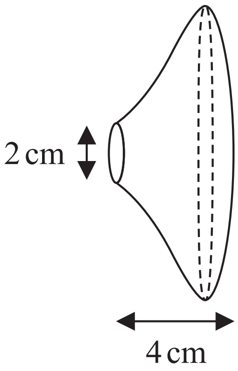

Figure 3

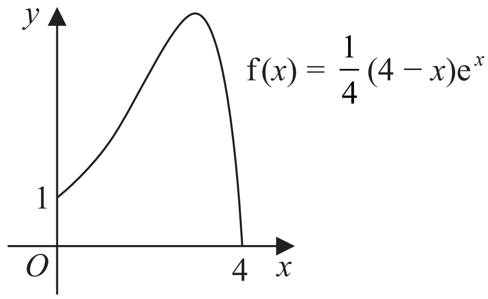

Figure 4

Figure 3 shows the design of a doorknob.

The shape of the doorknob is formed by rotating the curve shown in Figure 4 through 360° about the x-axis, where the units are centimetres.

The equation of the curve is given by

$$f(x) = \frac{1}{4}(4 - x)e^x \qquad 0 \leqslant x \leqslant 4$$

(a) Show that the volume, V cm³, of the doorknob is given by

$$V = K\int_0^4 (x^2 - 8x + 16)e^{2x}\,dx$$

where K is a constant to be found.

(3)

(b) Hence, find the exact value of the volume of the doorknob.

Give your answer in the form $p\pi(e^q + r)$ cm³ where p, q and r are simplified rational numbers to be found.

(5)

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Question 7 continued

(a) Formula for Volume of Revolution around the x-axis:

$$V = \pi \int_a^b y^2 \, dx$$

where a and b are the points on the x-axis bounding the area

$$f(x) = \frac{1}{4}(4-x)e^x$$

$$V = \pi \int_0^4 \left(\frac{1}{4}(4-x)e^x\right)^2 dx \quad \text{M1}$$

$$= \pi \int_0^4 \frac{1}{16}(4-x)^2 e^{2x} \, dx \quad \text{apply power}$$

M1

$$V = \frac{1}{16}\pi \int_0^4 (16 - 8x + x^2) e^{2x} \, dx \quad \text{take out constant, expand bracket}$$

hence shown A1

(b)

$$V = \frac{1}{16}\pi \int_0^4 (16 - 8x + x^2) e^{2x} \, dx \quad \text{multiplication} \therefore \text{by parts}$$

$$\int uv' \, dx = uv - \int u'v \, dx$$

$$u = x^2 - 8x + 16 \xrightarrow{\frac{d}{dx}} u' = 2x - 8$$

$$v = \frac{e^{2x}}{2} \xleftarrow{\int} v' = e^{2x}$$

$$= \frac{\pi}{16}\left[\frac{e^{2x}}{2}(x^2 - 8x + 16) - \int_0^4 (2x-8) \cdot \frac{e^{2x}}{2} \, dx\right]_0^4 \quad \text{by parts again}$$

M1A1

$$u = 2x - 8 \xrightarrow{\frac{d}{dx}} u' = 2$$

$$v = \frac{e^{2x}}{4} \xleftarrow{\int} v' = \frac{e^{2x}}{2}$$

$$= \frac{\pi}{16}\left[\frac{e^{2x}}{2}(x^2 - 8x + 16) - \left((2x-8)\left(\frac{e^{2x}}{4}\right) - \int_0^4 2 \cdot \frac{e^{2x}}{4} \, dx\right)\right]_0^4$$

$$= \frac{\pi}{16}\left[\frac{e^{2x}}{2}(x^2 - 8x + 16) - (x-4)\left(\frac{e^{2x}}{2}\right) + \frac{e^{2x}}{4}\right]_0^4 \quad \text{dM1}$$

$$= \frac{\pi}{16}\left[\left(\frac{e^{2(4)}}{2}\cancel{(4^2 - 8(4) + 16)}^{0} - \cancel{(4-4)}^{0}\left(\frac{e^{2 \cdot 4}}{2}\right) + \frac{e^8}{4}\right) - \left(\frac{\cancel{e^{2(0)}}^{1}}{2}(0 - 8(0) + 16) - (-4)\left(\frac{\cancel{e^{2(0)}}^{1}}{2}\right) + \frac{\cancel{e^{2(0)}}^{1}}{4}\right)\right]$$

$$= \frac{\pi}{16}\left(\frac{e^8}{4} - \frac{1}{2}(16) - \frac{1}{2}(4) - \frac{1}{4}\right) \quad \text{ddM1}$$

$$= \frac{\pi}{16}\left(\frac{e^8}{4} - 8 - 2 - \frac{1}{4}\right)$$

$$= \frac{\pi}{64}(e^8 - 41) \text{ cm}^3 \quad \text{A1}$$

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Question 7 continued

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Question 7 continued

Q7

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8. With respect to a fixed origin O the points A and B have position vectors

$$\begin{pmatrix}6\\6\\2\end{pmatrix} \text{ and } \begin{pmatrix}6\\0\\7\end{pmatrix}$$

respectively.

The line l_1 passes through the points A and B.

(a) Write down an equation for l_1

Give your answer in the form $\mathbf{r} = \mathbf{p} + \lambda\mathbf{q}$, where λ is a scalar parameter.

(2)

The line l_2 has equation

$$\mathbf{r} = \begin{pmatrix}3\\1\\4\end{pmatrix} + \mu\begin{pmatrix}1\\5\\9\end{pmatrix}$$

where μ is a scalar parameter.

(b) Show that l_1 and l_2 do **not** meet.

(4)

The point C is on l_2 where $\mu = -1$

(c) Find the acute angle between AC and l_2

Give your answer in degrees to one decimal place.

(5)

(a) Formula for vector line:

$r =$ position vector $+ \lambda$(direction vector)

Where the position vector is any point on the line and direction vector is a vector parallel to the line.

Get direction vector:

$\vec{AB} = \vec{OB} - \vec{OA}$

$= \begin{pmatrix}6\\0\\7\end{pmatrix} - \begin{pmatrix}6\\6\\2\end{pmatrix}$ M1

$= \begin{pmatrix}0\\-6\\5\end{pmatrix}$

Hence vector line in format $r = \vec{OA} + \lambda\vec{AB}$:

$\therefore r = \begin{pmatrix}6\\6\\2\end{pmatrix} + \lambda\begin{pmatrix}0\\-6\\5\end{pmatrix}$ A1

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Question 8 continued

(b) Equate the general points of l_1 and l_2. If there are values μ and λ which satisfy all 3 components, the lines intersect.

$$\begin{pmatrix} 6 \\ 6-6\lambda \\ 2+5\lambda \end{pmatrix} = \begin{pmatrix} 3+\mu \\ 1+5\mu \\ 4+9\mu \end{pmatrix}$$ M1

Solve x to get value for μ:

$$6 = 3+\mu$$

$$\mu = 3$$ A1

Use y to get value for λ:

$$6-6\lambda = 1+5(3)$$

$$-6\lambda = 10$$

$$\lambda = -\frac{10}{6} = -\frac{5}{3}$$ M1

Use z to get another value for λ to check if it is also $-\frac{5}{3}$.

$$2+5\lambda = 4+9(3)$$

$$2+5\lambda = 31$$

$$\lambda = \frac{29}{5}$$

The values of λ do not agree, hence the lines don't meet. A1

(c) We need to find $\vec{OC}$ in order to get $\vec{AC}$.

$$\vec{OC} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} + (-1)\begin{pmatrix} 1 \\ 5 \\ 9 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -5 \end{pmatrix}$$ M1

Hence get $\vec{AC}$:

$$\vec{AC} = \vec{OC} - \vec{OA}$$

$$= \begin{pmatrix} 2 \\ -4 \\ -5 \end{pmatrix} - \begin{pmatrix} 6 \\ 6 \\ 2 \end{pmatrix} = \begin{pmatrix} -4 \\ -10 \\ -7 \end{pmatrix}$$ dM1

We need angle between $\vec{AC}$ and l_2:

Use Formula:

$$\cos\theta = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|}$$ Formula for dot product: $\begin{pmatrix} a \\ b \\ c \end{pmatrix}\cdot\begin{pmatrix} d \\ e \\ f \end{pmatrix} = ad+be+cf$

Use $\vec{AC}$ and the direction vector of l_2:

$$\cos\theta = \left|\frac{\begin{pmatrix} -4 \\ -10 \\ -7 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ 5 \\ 9 \end{pmatrix}}{\sqrt{4^2+10^2+7^2}\,\sqrt{1^2+5^2+9^2}}\right|$$ ddM1 A1

$$\theta = \cos^{-1}\left|\frac{-4-50-63}{\sqrt{165}\,\sqrt{107}}\right|$$ plug this into your calculator

$$\theta = 28.3^\circ$$ A1

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Question 8 continued

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Question 8 continued

Q8

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9. (a) Find the derivative with respect to y of

$$\frac{1}{(1+2\ln y)^2}$$

(2)

(b) Hence find a general solution to the differential equation

$$3\operatorname{cosec}(2x)\frac{\mathrm{d}y}{\mathrm{d}x} = y(1+2\ln y)^3 \qquad y > 0 \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

(4)

(c) Show that the particular solution of this differential equation for which $y = 1$ at $x = \frac{\pi}{6}$ is given by

$$y = \mathrm{e}^{A\sec x - \frac{1}{2}}$$

where A is an irrational number to be found.

(5)

(a) $\frac{d}{dy}(1+2\ln y)^{-2}$

$= -2(1+2\ln y)^{-3} \cdot \frac{2}{y}$ chain rule: multiply by the derivative of the bracket M1

$= \frac{-4}{y(1+2\ln y)^3}$ A1

(b) $3\operatorname{cosec}(2x)\frac{dy}{dx} = y(1+\ln y)^3$

$\frac{3}{\sin 2x}\frac{dy}{dx} = y(1+\ln y)^3$

$\int \frac{1}{y(1+\ln y)^3}\,dy = \int \frac{1}{3}\sin 2x\,dx$ Separate variables by grouping the like terms on one side

Manipulate this to get the given expression from (a)

$\int -\frac{1}{4}\left(\frac{-4}{y(1+2\ln y)^3}\right)dy = \int \frac{1}{3}\sin 2x\,dx$ M1

$-\frac{1}{4}\left(\frac{1}{(1+2\ln y)^2}\right) = -\frac{1}{6}\cos 2x + c$ M1A1

$\Rightarrow -\frac{1}{4(1+2\ln y)^2} = -\frac{1}{6}\cos 2x + c$ A1

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Question 9 continued

(c) Substitute $y=1$, $x=\frac{\pi}{6}$:

$$-\frac{1}{4(1+2\ln 1)^2} = -\frac{1}{6}\cos\frac{\pi}{3} + c$$

$$-\frac{1}{4} = -\frac{1}{12} + c$$

$$c = -\frac{1}{6} \quad \text{M1}$$

$$\therefore -\frac{1}{4(1+2\ln y)^2} = -\frac{1}{6}\cos 2x - \frac{1}{6} \quad \text{A1}$$

Now we need to make y the subject:

$$\frac{1}{4(1+2\ln y)^2} = \frac{1}{6}\cos 2x + \frac{1}{6}$$ multiply both sides by -1

$$\frac{3}{4(1+2\ln y)^2} = \frac{1}{2}\cos 2x + \frac{1}{2}$$ multiply both sides by 3

double angle formula:

$\cos 2x = 2\cos^2 x - 1$

$\therefore \cos^2 x = \frac{1}{2}\cos 2x + \frac{1}{2}$

$$\frac{3}{4(1+2\ln y)^2} = \cos^2 x \quad \text{M1}$$

$$\frac{3}{4\cos^2 x} = (1+2\ln y)^2$$

$$\frac{3}{4}\sec^2 x = (1+2\ln y)^2$$ $\frac{1}{\cos^2 x} = \sec^2 x$

$$\sqrt{\frac{3}{4}\sec^2 x} = 1 + 2\ln y$$

$$\Rightarrow \ln y = \frac{1}{2}\left(\frac{\sqrt{3}}{2}\sec x - 1\right) \quad \text{ddM1}$$

$$e^{\ln y} = e^{\frac{1}{2}\left(\frac{\sqrt{3}}{2}\sec x - 1\right)}$$ $e^{\ln x} = x$.

$$\Rightarrow y = e^{\frac{\sqrt{3}}{4}\sec x - \frac{1}{2}} \quad \text{A1}$$

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Question 9 continued

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Question 9 continued

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Question 9 continued

Q9

(Total 11 marks)

TOTAL FOR PAPER IS 75 MARKS

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